ERC Starting Grant 2021
Research proposal [Part B1]
(Part B1 is evaluated both in Step 1 and Step 2,
Part B2 is evaluated in Step 2 only)

Resolving cosmological tensions with diverse data, novel theories and Bayesian machine learning

COSMOTENSION

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Proposal duration: 60 months

Our Universe is expanding faster than we expected. The standard model of cosmology predicts a Hubble constant which differs substantially from what we measure it to be. Far from being a problem, this is exciting as it indicates the need for a new model of the Universe. Despite three years of effort however, cosmologists have been unable to resolve this cosmic conundrum.

The reasons for this community failure are threefold. First, no single theoretical solution is capable of satisfactorily resolving all discrepancies. Second, disentangling new physics from measurement error is a challenging unsolved problem. Third, our simulation and data analysis pipelines have been designed and tuned in the context of the standard model, which can bias even the most carefully designed approach.

This ambitious project proposes to resolve all three of the above and uncover and establish the next cosmological paradigm for theory and data analysis. An interlocking programme of theory, inference and observational research, undertaken by the PI, three postdocs and four PhD students over five years will aim to simultaneously resolve the tensions in both cosmological theories and data processing.

The broad aims of the project are to (a) Resolve the tensions between cosmological observations with a new standard model of the universe and next-generation numerical techniques (b) Establish likelihood-free inference at the heart of our cosmological analysis toolkit in preparation for the future onslaught of big cosmological data, and (c) Bring together a diverse set of cosmological and particle physics datasets and organise them in a coherent statistical framework.

This is an essential and substantial research effort which only an ERC starting grant can support.

N/A

Section a: Extended Synopsis of the scientific proposal

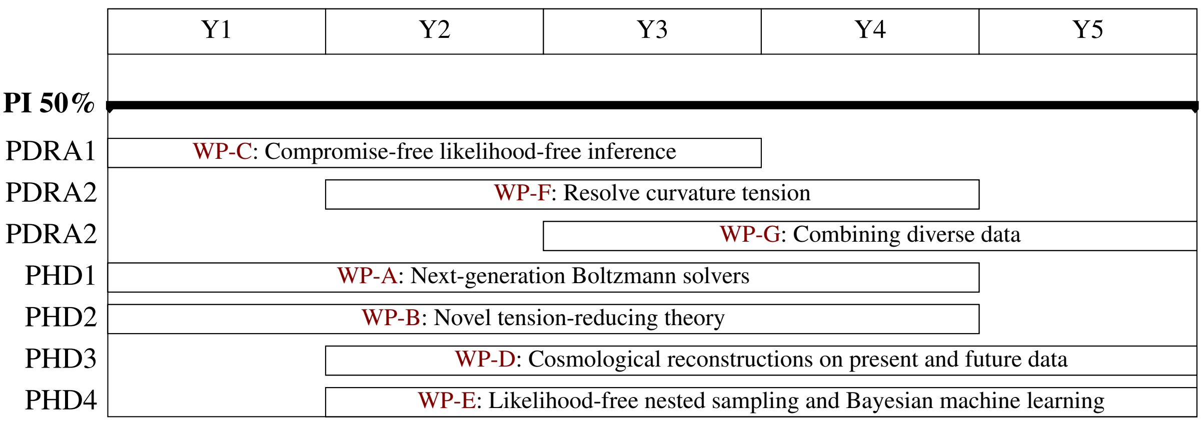
Why is our Universe expanding so fast? Around this question, a crisis is brewing in precision cosmology. In 2013 the Planck satellite and its team [1] detailed in exquisite precision its measurements of the cosmic microwave background, and delivered an outstanding confirmation of our "standard model of the Universe" ΛCDM [2]. However, since then observations using other cosmological phenomena such as supernovae measurements [3] and large scale structure surveys [4, 5] have been coming to different conclusions. Their datasets are consistent with different values of fundamental cosmological parameters, such as the present day expansion rate H_0 [6], the amount of matter clustering in the Universe S_8 [7], or the shape of the Universe Ω_K [8]. Far from being a disaster, such discrepancies are scientifically exciting. The last time this happened in cosmology in the early 1990s we were forced to introduce "Dark Energy" into our standard model. The cosmological community agree [9] that the time is ripe for the next evolution of our theories and a move to a new standard model of our Universe.

However, unlike three decades ago when the discrepancies in the present day acceleration rate of the universe were resolved with the introduction of dark energy, after nearly three years of community effort, no resolution this time is forthcoming. The reasons for this are threefold: *First*, despite exhaustive searches there is no simple change to the standard model using known or speculative physics which is capable of satisfactorily explaining all the discrepancies. *Second*, the datasets are of significantly higher volume and quality than thirty years ago, with sophisticated data analysis pipelines required to process them. Disentangling discrepancies associated with systematic errors in data gathering from new physics is a largely unsolved problem and hampers efforts to isolate the cause of and solution to these inconsistencies. *Third*, all of our sophisticated computational physics pipelines have been developed and tuned in the context of our standard model, which can bias even the best-designed systematics analyses.

My research proposal aims to address all of these issues, with an interlocking programme of theory, observational data analysis and computational method development for which I am especially skilled and situated and capable of leading. It is an ambitious project, with a large and dedicated team of myself, three 3-year postdoctoral research associate positions (PDRA) and four 4-year PhD students with a diverse set of skills. The primary goal is nothing further than a more complete understanding of our Universe.

The proposal is structured into seven work packages, one for each of the PDRAs and PhD students (diagram below). As PI, I will oversee and contribute to all, and there is substantial cross-talk between work packages. The project is structured with a symmetric ramp-up and down so that at its peak in Y3 all eight members are present and maximally skilled. Placing the peak at the centre increases the likelihood of success within five years of this high-risk high-reward proposal, even accounting for inevitable error bars on timelines. The University of Cambridge is a particularly suitable place to undertake this project; As a world-class institution for theory, observation and inference in astrophysics it attracts many of the strongest PhD and PDRA applicants and hosts an anomalously large community of astrophysicists with viewpoints which span the range of interpretations in the global cosmological community.

This ambitious research programme which seeks to restructure the fundamentals of how we analyse data and think about theory is only capable of being funded by an ERC starting grant. It is my view that unless such a programme is supported, and these bedrock issues are addressed, cosmologists run the risk of confirmation bias as the next generation of gigantic datasets come crashing down on us.



Background

The standard model of cosmology (ΛCDM) describes the large scale structure, composition and evolution of our Universe with a stunningly compact set of only six parameters. The implications and predictions of this model percolate down into nearly every astronomical analysis. Since the 1990s, it has risen as the model which best explains the now extremely wide variety of cosmological observations.

Despite its phenomenal phenomenological success, there are many drawbacks associated with the standard model. As soon as one is forced to spell out its details, like the camel it has an intangible quality of design by committee. It is also somewhat embarrassing to explain the name "ΛCDM" to a lay audience, since the acronym describes the two invisible components (dark energy and cold dark matter) which we have to inject in order to make our equations and observations add up.

Moreover, the past thirty years of observations have taken their toll, and our data are now statistically sharp enough to reveal the cracks at the heart of the modelling. Measurements of the expansion rate H_0 differ substantially between cosmologists who use nearby supernovae measurements and those who infer its value today from observations of the cosmic microwave background (CMB). Measurements of the degree of matter clustering S_8 differ between weak lensing and CMB observations, and conclusions regarding the shape of the universe quantified by spatial curvature Ω_K remain divisive [10–12].

It is a feature of the history of physics that small discrepancies in an otherwise powerful framework can spark a revolution in theory, as exemplified by the photoelectric effect for the discovery of quantum mechanics, or mercurial perihelion shift for general relativity. It is not impossible that resolving these cracks in our standard model may reveal solutions to deeper tensions in physics, such as the incompatibility of quantum mechanics and gravity, and the mysterious role that thermodynamics seems to play at their interface [13].

With all of these incentives and the community acceptance that there is a problem, it is in some sense remarkable that no widely accepted solution has been forthcoming. The thesis of this project is that it is probable we need to modify more than one component of the standard model (for example removing dark energy, and replacing it with better motivated physics) rather than simply seeking extensions. Before doing this however we must fix deeper issues seated at the heart of cosmological analysis.

Strategy

This proposal seeks to "disentangle new physics from systematics" by first permanently removing the deep-seated biases at the heart of our cosmological data analysis pipelines, and then following this up with the state-of-the art in theoretical modelling and Bayesian inference.

The first of these biases is that at the centre of any cosmological analysis involving the cosmic microwave background (our gold-standard dataset) are fine-tuned Boltzmann solvers. By necessity, these are designed to be lean and fast in the context of the standard model, but once one strays off the beaten path into extensions they become slower and less accurate. This has two issues. The first is that while this potential and unquantified bias remains, it casts a shadow over the true relative consistency of alterations to ΛCDM. The second is more sociological, in that so long as it is an order of magnitude harder to test a new theory from scratch in comparison to the baseline, there will always be an additional erroneous resistance to change. In this proposal I aim to fix this "code debt" permanently by developing RKWKB Boltzmann solvers.

The second is that at the heart of any inference approach (Bayesian or Frequentist) lies the likelihood function. These are possible (though not easy) to compute directly for a cosmic microwave background analysis. The future of cosmology will use observations of more evolved objects such as weak lensing, baryon acoustic oscillations and supernovae. In these instances the likelihood is impossible to write down, and any analytic assumption one makes can subtly bias the results. In this proposal, we will establish the principles of likelihood-free inference at the heart of future pipelines through their coherent application to a suite of present day datasets.

Alongside these substantial changes to our data analysis, the project will develop novel theories alongside existing extensions, as well as using model-independent reconstruction techniques. The project therefore begins in Y1 with three work packages aimed at Theory, LFI and Boltzmann solvers. In Y2, we begin applying these techniques to the curvature tension, bring further resource and research into more speculative long-term enhancements to LFI, as well as complementing this strategy with research into model-independent cosmological reconstructions. In Y3, the focal point of the project, the final PDRA joins, and we will begin a first attempt at resolving tensions using all the techniques in tandem. PDRA3 will remain until the end of the project, refining and iterating the procedures, as other projects transition to more forward-looking elements and forecasts.

Objectives

- (O1) Develop an RKWKB Boltzmann solver
- (O2) Distribute the community-standard LFI tools and code
- (O3) Create and develop ambitious tension-resolving theories
- (O4) Consistently and coherently test alternatives to ΛCDM
- (O5) Resolve the curvature tension
- (O6) Resolve the Hubble tension

Deliverables

- (D1) Multivariate RKWKB code
- (D2) Next-generation Boltzmann software
- (D3) LFI cosmological software packages
- (D4) Cosmological reconstruction framework
- (D5) Transdimensional & LF nested sampling
- (D6) A new model of the Universe

Dissemination

Other than the potential for a solution to (m)any of the cosmological tensions, this project will leave a legacy of techniques and software suitable for future generations of astrophysical researchers. I have already fostered a culture within my PhD students of industry-standard open source software distribution using git(hub) version control, continuous integration, and pip-installable Python packages, for example `maxsmooth` [14, 15], `globalemu` [16], `anesthetic` [17], `oscode` [18, 19]. I also maintain the highest data management practices, ensuring that with any publication, the data, code and plotting scripts are reproducibly made available on Zenodo [20–22]. This research hygiene ensures the products are both available to and able to be challenged by future researchers (including most importantly ourselves). As with all my research, papers produced by the project will be made openly available for free on arχiv, in addition to being published in a range of journals such as MNRAS, PRD, JCAP, JHEP, Nature (astronomy) and PRL. I expect at least one paper per sub-objective (i.e. more than thirty), and we will further publicise our work at seminars, workshops and conferences.

Risk mitigation

- A significant risk is that we may be unable to find a solution without future datasets. However, even in the event that we do not reveal a successor cosmology to ΛCDM, the techniques, theories and software we build will still provide critical tools otherwise unavailable for future researchers for resolving this most fundamental of cosmological issues. The later stages of the project focus on forecasts to partially mitigate this risk.
- In the event that the computing power requested is not sufficient to carry out some of our more expensive analyses, we can obtain time from GAMBIT community resources, from the DiRAC resource allocation I currently hold, and apply for future follow-on DiRAC resource.
- Another "risk" is that the project could reveal that all the current tensions are explained by systematics rather than new physics. Whilst this would be scientifically disappointing, this would still be a significant step forward for the community, allowing us to proceed on a solid standard model foundation as more statistically powerful datasets are acquired.

WP-A Next-generation Boltzmann solvers [PHD1]

This work package will replace the Runge–Kutta-based approach at the heart of existing Boltzmann codes with multivariate RKWKB techniques [19, 23, 24], developed over the course of the project (O1). The current state of the art CAMB [25] and CLASS [26] have several ΛCDM-specific approximations [27], and can be viewed as finely-tuned tuned Goldberg engines, with phenomenal speed and precision at solving cosmologies in the context of the standard model. This work package aims to increase the speed and accuracy of the fundamental algorithm to the extent that all modes can be solved for without the need for approximations or shortcuts, and allow extensions to be simulated on the same footing as the standard model. It will contribute to (D1) and (D2).

- (OA1) Extend theory and state of the art to include forced equations.
- (OA2) Extend theory and state of the art to include multivariate RKWKB
- (OA3) Develop performant, publicly distributable code for solving multivariate differential equations
- (OA4) Build Boltzmann solver on top of this, demonstrating precision adjusted speed-up on standard model
- (OA5) Apply this solver to extensions such as curvature and neutrinos, demonstrating equivalent speed up
- (OA6) Incorporate this code into community tools such as GAMBIT and cobaya

WP-B Novel tension-reducing theory [PHD2]

This work package trains a dedicated team theorist in time for Y3, capable of developing novel theories for solving cosmological tensions (O3) and reacting to new developments in theoretical modelling. I have selected three novel lines of attack which I believe show promise in resolving multiple tensions in cosmology. First Poincaré gauge theory gravities [28–30] have been shown to simultaneously explain both dark matter, dark energy and the Hubble discrepancy using a modified form of gravity which also shows promise in terms of renormalisability. Second, investigations into initial conditions for inflation [31–33], which strikes at the heart of tensions in quantum gravity and in late time curvature. Third, future conformal boundary theories [24, 34–36], which unify holographic principles with Lasenby's de Sitter space models [37], and which will require significant interaction with PHD1 for deriving predictions from these theories. The student and rest of the team will however remain reactive to community developments and be prepared to pivot if novel tension-resolving theories reveal themselves. This will contribute to (D2).

(OB1) Familiarise with current state of the art in tension resolving theories
(OB2) Develop and apply perturbation theory for Poincaré gauge theory gravities
(OB3) Investigate initial conditions for inflation tension
(OB4) Develop future conformal boundary theory for tension resolution

WP-C Compromise-free likelihood free inference [PDRA1]

This work package applies my work on Bayesian sparse reconstruction [38] to the field of likelihood-free inference (LFI), with the primary goal of producing a form of LFI which is robust, extendable and trustworthy in the long term (O2). It frees the existing state-of-the-art (DELFI [39] and BOLFI [40]) from neural network and Gaussian process based approaches, replacing optimisation of hyperparameters with Bayesian marginalisation/sampling. After an initial framework is prepared, this strategy will be battle-tested on CMB, BAO, SNe, WL and CMB data. The aim is to both verify or contest existing analyses whilst simultaneously demonstrating the wide scope and applicability of the fundamental techniques. Results will be distributed as a robust software package (D3).

(OC1) Release a reusable code for compromise-free likelihood-free inference
(OC2) apply CFLFI to CMB lensing and compare to the standard result
(OC3) apply CFLFI to BAO and compare to the standard result
(OC4) apply CFLFI to Supernovae data and compare to the standard result
(OC5) apply CFLFI to weak lensing and compare to the standard result
(OC6) apply CFLFI to CMB and compare to the standard result

WP-D Cosmological reconstructions on present and future data [PHD3]

This work package aims to complement the theoretical work in WP-B and contribute to (O3) by instead reconstructing critical portions of the cosmological pipeline in a "model independent" or non-parametric way. This builds on my landmark work in primordial power spectrum reconstruction for the Planck satellite [41], beginning by applying this fully Bayesian strategy to a wider set of cosmological quantities such as the reionisation history and CMB power spectrum. Later elements of the package will combine my tension quantification techniques such as the Suspiciousness statistic [42, 43] to create a "functional tension quantification" setup, and analyse whether datasets are consistent at a model independent level. The final stages of the project involve simultaneous reconstruction, and an extension of the nested sampling framework to include transdimensional navigation (D5), which will prove essential for simultaneously reconstructing multiple cosmic histories. These developments will be distributed to the community in a dedicated and reusable software package (D4).

(OD1) Develop a generalised reconstruction pipeline
(OD2) Reconstruct separate elements of cosmic history using present day data
(OD3) Use functional tension quantification techniques to determine where tensions reside
(OD4) Develop transdimensional nested sampling in the context of reconstructions
(OD5) Apply transdimensional nested sampling to simultaneous reconstruction
(OD6) Determine the future limits and potential of these techniques on forecast data

WP-E Likelihood-free nested sampling and Bayesian machine learning [PHD4]

This work package will complement WP-C and contribute to (O2). It takes a more forward-looking view more appropriate to a PhD student project than just the goals of Y3, and further development of (D3) and (D5). It will aim to more tightly integrate nested sampling and likelihood free inference, building on the work of [44], and addressing the critical curses of dimensionality associated with density estimation in classic nested sampling style. In addition, it will build on the work of Heavens [45, 46] in extreme data compression techniques applying the high dimensional toolkit available to nested samplers, and look to join the emerging field of topological data analysis [47, 48] with LFI. Towards the end of the project, it will look into applying and developing techniques in accelerated nested sampling to the high-dimensional problems found throughout this project.

(OE1) Bring likelihood free inference + nested sampling up to the state of the art in both
(OE2) Develop Bayesian Kullback–Leibler-based simultaneous compression statistics
(OE3) Explore the use of Topological data analysis in LFI compression
(OE4) Investigate the use of Bayesian Neural Networks (`PolyNet`) as a Bayesian neural density estimator
(OE5) Use accelerated nested sampling for higher-dimensional CFLFI
(OE6) Release next-generation version of LFI for cosmology as codebase

WP-F Resolve curvature tension [PDRA2]

This work package addresses (O5) and provides the bedrock of (O4) to contribute to (D6). It aims to resolve one of the major tensions in both the data and the community by explaining the curvature tension. It will also act as a case study in applying Boltzmann codes and LFI in preparation for the more substantial final work package. The key issue it aims to resolve is that degeneracy breaking likelihoods such as BAO and CMB lensing have fiducial flatness assumptions at the heart of their calculations. Whilst these remain, curved-universe sceptics will point to the fact that these land analyses with curvature at a result with Ω_k suspiciously consistent with exactly zero. These will be removed using likelihood free inference to jump straight from forward simulation to likelihood, without any need for calibrations associated with flat cosmologies or expansions about a Gaussian likelihood. The aim here is to close the book on the debate. If it is discovered that in fact curved models are preferred by an unbiased BAO treatment, then this has profound implications for the project and the rest of cosmology, and some pivoting of other work packages surrounding this will be required. If it is not, then this is a smoking gun that our final concordance model must be capable of emulating curvature-like effects (such as A_L) in the cosmic microwave background but not in late-time datasets.

(OF1) Remove residual flat assumptions from baryon acoustic oscillation pipeline
(OF2) Remove residual flat assumptions from CMB lensing pipeline
(OF3) Test CMB, BAO and lensing in the context of curvature
(OF4) Investigate curvature $\pm X$ models

WP-G Combining diverse data [PDRA3]

This work package aims to address (O4) & (O6) and deliver (D6). It begins at the heart of the project in Y3, when all members are present and fully skilled. The likelihood-free inference products from WP-C and WP-E will be used for disentangling systematics. The Boltzmann codes from WP-A will mean that we are capable of testing extensions without standard model bias. The novel combination of theories proposed by WP-B and WP-F will form our universe of models for explaining the tension, which will in turn have been informed and crafted by the model independent analyses of WP-D. After a first attempt at resolving the Hubble tension, this will be iterated on. Depending on the degree of success, more diverse datasets will be brought to bear on the analysis, such as 21-cm global data, and particle physics modelling. Toward the end of the project, as the work packages become more forward-looking, the focus will shift to forecasted data for instruments such as Euclid, SKA and CMBS4.

(OG1) Synthesise all prior work package products into a coherent statistical framework
(OG2) Taking a global view, determine which (if any) model is capable of consistently fitting all the data.
(OG3) Incorporate GAMBIT particle physics data for constraining neutrino and dark matter properties.
(OG4) Use REACH 21-cm data for improved τ constraints
(OG5) Apply the same framework to forecasted data

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ERC Starting Grant 2021
Part B2

Section a: State-of-the-art

The era of precision cosmology was heralded by the first high-precision mapping of the anisotropies in the cosmic microwave background by the WMAP [1] and Planck [2] satellites. As a cosmologist I was academically born in the epoch of precision cosmology, with the first Planck data release [3] occurring in the sixth month of my PhD. My research career has therefore been one of watching and subjecting a triumphal standard model of the universe (ΛCDM) to steadily more and more stringent tests from a variety of different astronomical observations. For a significant fraction of these tests, the standard model has now been found wanting. Despite several years of attempting to resolve these discrepancies or "tensions" however, the cosmological community has failed to find a satisfactory explanation. It is the purpose of this ambitious starting grant to simultaneously resolve the critical issues with our current scientific analyses, set the paradigm for the next generation of cosmological data science and to uncover the next standard model of the Universe.

The standard model of cosmology

There are many ways of defining a cosmological model of our Universe, but in short they must describe (a) its material composition (b) its size and shape, and (c) its initial conditions and evolution. The standard model ΛCDM achieves this in just six parameters [4]. It assumes a universe whose large-scale evolution is dictated by Einstein's theory of general relativity, where constituents (matter, dark matter and radiation) are distributed on the largest scales homogeneously and isotropically. There are six *a priori* unknown quantities: Three for its constituents (the fraction of visible/"baryonic" matter Ω_b, the fraction of slowly moving "cold" dark matter Ω_c, and an optical depth to reionisation τ), one for its scale and shape (a spatially flat universe with present day Hubble parameter H_0), and two for the initial state phenomenologically described by a primordial power spectrum with amplitude A_s and tilt n_s. In order to achieve a spatially flat universe, the model must also introduce a large quantity of dark energy.

There are of course many ways of phrasing these parameters, and in fact CMB cosmologists for both physical and sampling convenience usually use a less degenerate combination of Hubble-rescaled equivalents of the baryonic and dark matter fractions, and a rescaled angular recombination sound horizon size θ_{MC} in place of the Hubble parameter H_0. These choices are to a large extent equivalent, and the six parameters chosen are often more for computational convenience than philosophical compactness (although if one is not careful this can introduce unexpected parameter volume/prior effects into an analysis).

Within this framework, it is easy to propose new extensions and alternatives to ΛCDM. Changing the composition could involve (but is not limited to [5]) a more sophisticated time and spatially varying model of dark energy w [6], more or less active and detailed neutrino content ν [7] or interacting dark matter and "dark radiation" [8]. Changing the geometry could involve a relaxation of the flatness assumption $\Omega_k \neq 0$ [9], a degree of large scale anisotropy [10] or inhomogeneity (Hubble bubble [11]). Changing the initial conditions and evolution can involve sophisticated primordial models of the Universe (inflation) or testing modifications or alternatives to Einstein's theory of gravity. Testing these extensions and alternatives is however another story.

Tensions in our standard model

The need for extensions has been drawn to the community's attention by the now quite radically different values which different datasets yield for the Hubble parameter H_0, but in reality there are a host of long-standing tensions between datasets, within our theories, and at the heart of our data analysis.

We begin with a few of the tensions between datasets, represented in Figure 1. First, the **Hubble tension** is exemplified at its extreme by the fact that measurements from the standard type IA supernovae approach (which awarded Adam Riess the Nobel prize for the discovery of dark energy) reveal a Hubble parameter which is $H_0 = 74 \pm 1.4$, whilst the gold-standard Planck measurement yields $H_0 = 67.4 \pm 0.4$. More importantly, there is a general divide between measurements of properties of the nearby late-time universe, which sit toward the upper end of this scale, and inferences of the present day H_0 which rely on observations of further away and backward in time portions of the universe, which sit on the Planck end of the scale [12]. Second the **Weak lensing tension**, which is shown by the fact that measurements by the KiDS [13] and DES [14] team measure a

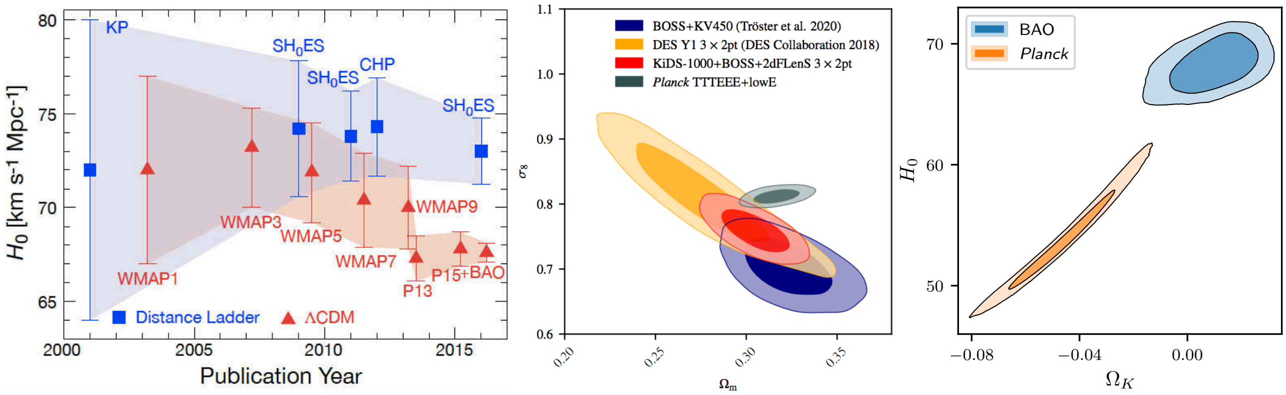

Figure 1: The Hubble, weak lensing and curvature tensions.

different combination (summarised by S_8) of the amount of matter in the universe Ω_m and matter clustering σ_8 than that inferred by Planck. Third, the **Lithium problem** arises from the fact that the observed abundance of Li in the Universe differs substantially from that required by ΛCDM for big bang nucleosynthesis [15]. Fourth, the **Curvature tension**, in which the Planck data have a moderate preference for closed universes, whilst baryon acoustic oscillations (BAO) have a resounding preference for flat ones [16, 17].

Within our theories, there are several long-standing tensions. The problem of **Initial conditions** asks why the universe is homogeneous and isotropic across distances so great that light would not have been able to traverse them over the history of the universe, and are thus causally disconnected [18]. This is to some extent explained by our semi-predictive theory of primordial inflation, although others argue that this merely moves the problem back in time, for example creating issues with measures on the multiverse [19]. Related but subtly distinct in the **Entropy tension**, which asks why the universe should have emerged in an anomalously low entropy state, or equivalently why the arrow of the evolution of the scale factor evolution should align with the thermodynamic arrow of time [20]. Largest of all in cosmology is however the astronomical elephant in the room: **Dark tension**. Our description of the universe only makes sense if ≈70% of it is comprised of invisible dark energy (whose non-zero value is theoretically embarrassingly low [21]), and another ≈25% dark matter, which does not fit into any of our standard models of particle physics, and we have yet to directly detect through any mechanism [22]. Worse, attempting to further investigate dark energy, we are drawn into "phantom" regions of the parameter space [23].

The largest tension of all however resides in the tension between quantum mechanics and gravity, in that we know these theories to be fundamentally mathematically incompatible, and therefore on some level incomplete. What is exciting about cosmology is that it gives us a laboratory in which we might be able to test the interface between the two [24, 25].

Bayesian cosmological data analysis

The detail of how one compares and contrasts models of the universe is performed in cosmology using Bayesian inference [26], the three pillars of which are:

Model comparison: How much do the data support a model?

Parameter estimation: What do the data tell us about the free and unknown parameters of a model?

Tension quantification: Do different datasets generate consistent predictions in the context of a model?

Consider a model M with parameters θ and data D — for a concrete example one can substitute $M =$ΛCDM, $\theta = (\Omega_b, \Omega_c, \tau, H_0, A_s, n_s)$ and D as the Planck CMB data. A cosmological analysis typically begins by writing down a generative "likelihood function" $L = P(D|\theta, M)$. The likelihood represents the forward probability of observing a given set of data D if you knew that M were ΛCDM, and some oracle gave you the "true" parameters θ of the universe. In reality writing the code for a likelihood function is a considerable amount of work, but the probabilistic calculation of the conditional probability L is something which both Bayesians and Frequentists agree on. Bayesians then go further to say that since we know the data D, but don't know if our model M is true, or what the parameters θ are, we should invert the likelihood to find the model probability

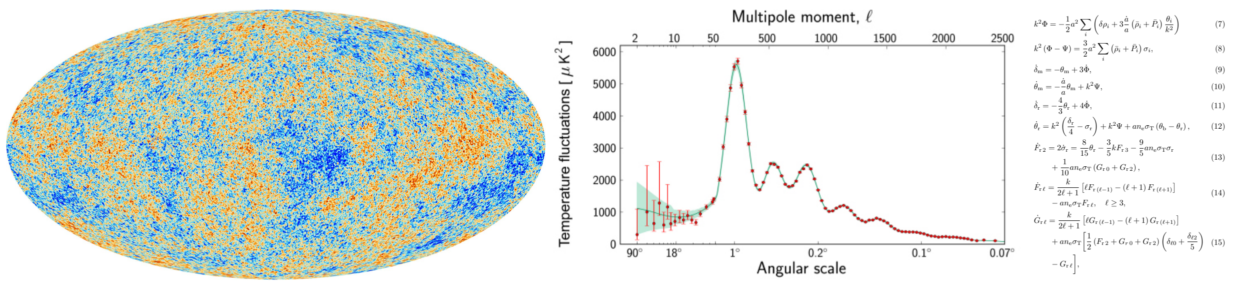

Figure 2: CMB anisotropy map and Boltzmann code predictions and theory equations

$P(M|D)$ [for model comparison] and posterior distribution $P(\theta|D,M)$ [for parameter estimation]. This can be performed using Bayes' theorems:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}, \qquad P(\theta|D,M) = \frac{P(D|M,\theta)P(\theta|M)}{P(D|M)} \tag{1}$$

providing one specifies a prior distribution over models $P(M)$ (usually taken to be uniform over a discrete set of choices) and a prior for the unknown parameters $P(\theta|M)$. In the same way that the likelihood $P(D|\theta,M)$ is the cornerstone of parameter estimation, the evidence $Z = P(D|M)$ (which features as a normalising constant in the second of Bayes theorems) is the cornerstone of both model comparison and tension quantification [26, 27]. A central feature of a Bayesian model comparison is that it mathematically quantifies the philosophical principle of Occam's razor, automatically preferring simpler models with equivalent explanatory power: $\log Z = \langle \log L \rangle - \mathscr{D}$ [28].

While parameter estimation can be performed numerically by a variety of Markov Chain Monte Carlo (MCMC) techniques such as Gibbs, Metropolis Hastings or Hamiltonian sampling, for model comparison choices are far more limited. Head-and-shoulders above any alternative in numerical evidence computation is Nested Sampling [29], a field for which I am a pioneer [30–37], which is capable of performing both model comparison and parameter estimation simultaneously. Tension quantification is a far newer field [38], but the techniques I have built [39–42] in general directly or indirectly have been enabled by and require nested sampling through either the need to compute the Bayesian evidence Z, or the need to navigate non-trivial parameter spaces when only a subset of the data are being used.

Boltzmann codes

The cosmic microwave background (CMB) is our gold-standard dataset, and will remain so for the next decade or more [2]. A CMB anisotropy map is effectively a pristine snapshot of the surface of last scattering: an image of the state of the universe approximately 300,000 years after it was born. This high-precision data has extreme statistical power which any model of the universe must adequately explain. The image on the left of Figure 2 shows small inhomogeneities which if wound forward in time would show clusters and galaxies coalescing around regions of high density, and if wound backward in time give us details of the quantum mechanical perturbations in the primordial universe.

This time-travelling viewpoint is made quantitatively possible through the use of Boltzmann codes, for which the current state of the art is held in CAMB [43] and CLASS [44]. Procedurally this science distils down into a set of non-linear ordinary differential equations for the background, and linear equivalents for the many perturbation modes and anisotropies. These differential equations are highly coupled, with each perturbation mode going through periods of slow variation and fast oscillation as the universe transitions between phases. Figure 2 shows this schematically, with the data on the left, the equations on the right, and Boltzmann codes allowing the link between the two to compare theory to compressed data in the middle.

If one only had to solve these equations once, then there would be no problem, since the numerical solution of differential equations is a relatively well-established field and solving the equations as-is only takes minutes to hours depending on the precision required. However, these solutions form part of the forward modelling procedure or "likelihood loop". These must therefore be evaluated thousands to millions of times in the process of Bayesian model fitting. In order to overcome this, many theoretically well-motivated approximations are deployed and numerical tricks (such as interpolation) applied [45]. This improves accuracy and speed by several orders of magnitude.

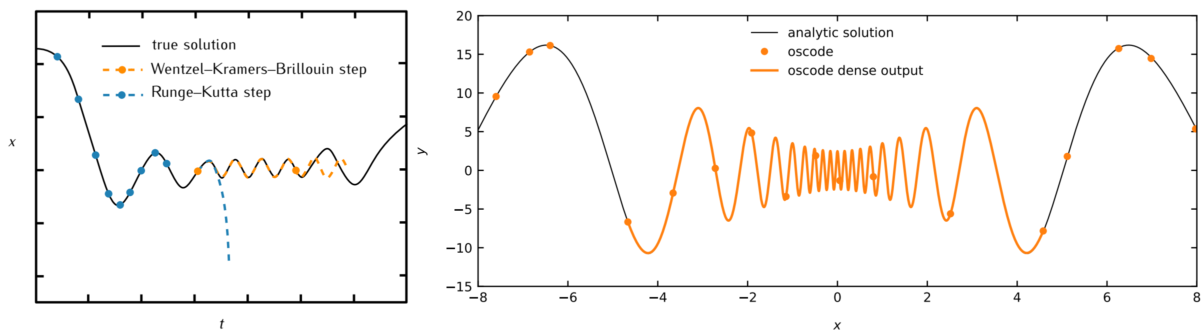

Figure 3: The Runge–Kutta–Wentzel–Kramers–Brillouin (RKWKB) method

These shortcuts are thoroughly understood and well-motivated, and their application enables cosmologists to incorporate this into a likelihood loop to extract the cosmological parameters presented in the Planck papers. Crucially however, this statement is only true for ΛCDM [25]. These approximations are not as effective for extensions (for example curvature or neutrinos), meaning testing these models comes with considerable computational cost and reduced accuracy. This "code debt" has two issues. The first is that while this potential and unquantified bias remains, it casts a shadow over the true relative consistency of extensions to ΛCDM with the data. The second is more sociological, in that so long as it is an order of magnitude harder to test a new theory from scratch in comparison to the baseline, there will always be an additional erroneous resistance to change.

Numerically solving differential equations should not be this hard. In the conventional sense it amounts to a discretisation of Taylor's theorem. This is done in an immensely clever way, so as to get simultaneous access to high orders of derivatives, but no amount of intelligent algorithm design will get round the fact that Taylor series are not good at approximating an oscillation, so Runge–Kutta-like approaches will have to trace every peak and trough, and it is here that the computational cost and accuracy issues set in. A key insight, shown in Figure 3, is to replace the Taylor approximation at the heart of Runge–Kutta approaches with a WKB approximation, which by definition will well-approximate an oscillating solution. This forms the RKWKB approach, which I invented in my PhD [46], and has been further developed and applied by myself and my students, both PhD [47, 48] and Masters [49, 50], and is now being adopted by other communities [51].

Likelihood-free inference, data compression and systematics

Likelihood-free inference (LFI) is an emerging paradigm in cosmology. In contrast to its name, far from removing likelihoods from the pipeline, it aims to solve the problem "How can we perform parameter estimation and model comparison when an exact likelihood is inaccessible or intractable?". LFI solves this problem providing that one can forward model or simulate the system. Such situations abound in modern cosmology, particularly when dealing with observations of the non-linear universe such as the matter power spectrum, galaxy clustering and large-scale structure. Traditionally in such situations, likelihoods are approximated with suspect Gaussianity assumptions, or expressed via a sequence of increasingly non-Gaussian terms (directing the field of research into nongaussianities [52]). LFI is still in its infancy, but is under active development and already producing publication-quality analyses, reproducing results when the likelihood is known, and finding new results when it is not [53]. It is my belief that LFI represents the future of inference, and we sit in an analogous position to fifty years ago at the birth of MCMC.

At its essence, LFI proposes to learn the likelihood from simulations. Assuming that from a given set of Universe parameters θ one can generate a set of mock data $\hat{D}$, one chooses a set of representative parameters $\{\theta_i, i=1\ldots N_\text{train}\}$, and goes through the following procedure:

$$\begin{gathered}\theta_i \xrightarrow{\text{simulate}} \hat{D}_i\\ w_\text{train} = \max_w \sum_i G[f(\hat{D}_i,\theta_i,w)],\end{gathered} \qquad f(D,\theta,w_\text{train}) = \begin{cases} P(D|\theta) & \Rightarrow \quad \text{Likelihood learning}\\ P(\theta|D) & \Rightarrow \quad \text{Posterior learning}\\ P(D,\theta) & \Rightarrow \quad \text{Joint learning.}\end{cases} \tag{2}$$

One first generates mock data for each set of parameters, and chooses a flexible proxy function $f(D,\theta,w)$ parameterised by w, and misfit functional G. Optimising this function over w using the generated data yields best-fit values w_train for the proxy parameters, which when put into f yield a function which approximates either

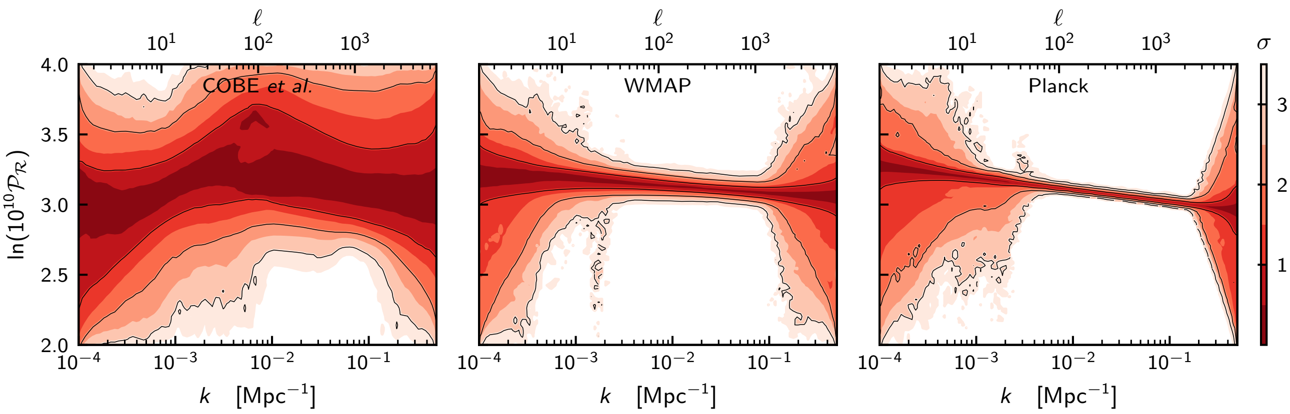

Figure 4: Model-independent reconstruction of the primordial power spectrum across human history

the true likelihood, the posterior or the joint distribution. Which of these the process produces is dependent on the choice of proxy f and misfit G, but in practice the misfit function is a least-squares or Kullback–Liebler-like construction. To avoid a curse of dimensionality, the data D usually need to be highly compressed relative to their raw forms.

The state of the art cosmologically is instantiated in DELFI [54], where f is a neural density estimator and BOLFI [55], where f is a Gaussian process. These represent a significant jump from the previous set of methods based around approximate Bayesian computation (ABC) [56]. Both methods depend on having access to a massively compressed dataset using techniques like MOPED [57] or their descendants [58].

LFI is a particularly powerful tool when it comes to detecting and modelling systematic errors [59]. From a Rumsfeltian perspective, a theory is our known known, the likelihood and error modelling are known unknowns, while systematic errors are unknown unknowns. A systematic is nothing more than a statistically unmodelled error, which can manifest as a bias in the results or incorrect spread in the residuals.

It's important to recognise that systematic modelling and detection is a delicate art, and LFI is not a silver bullet for removing systematics. What it does do however is free one from many of the constraints which afflict a traditional systematics treatment. Since all one needs to be able to do is forward model a systematic effect (rather than incorporating them into an explicit likelihood function), adding in new phenomena becomes extremely straightforward. The shift in focus toward concepts like the generative nature of likelihoods, and rendering explicit concepts like compression (which most physicists have internalised, but rarely articulate) means that proposing and testing systematics takes days rather than months.

Most importantly LFI is an orthogonal approach to likelihood construction, and a substantially more rapid one. Pipelines which took large teams years of effort in collaborations like Planck can be accomplished by a single person in months. If one finds that tensions remain unchanged upon an LFI re-implementation, it suggests the problem does not reside in the likelihood calculation or mistreatment of systematic errors.

Model independent reconstruction

Over the course of my research career, I have pioneered Bayesian reconstructions of the primordial power spectrum $\mathscr{P}_\mathscr{R}(k)$ from cosmic microwave background (CMB) data [60] (Figure 4), as well as applying it to the dark energy equation of state [6, 61, 62]. Under these non-parametric approaches, the data determine how much theoretical structure is supported, with the Bayesian evidence acting as the ultimate arbiter for the complexity of the fit [63]. The driving principle is that instead of a physically motivated (i.e. model-dependent) parameterisation of a function $f(x)$ of an independent variable x, a highly flexible form $f(x;\theta)$ is chosen with N spline-like parameters $\theta = \{(x_1,y_1),\ldots,(x_N,y_N)\}$ [64]. These parameters θ are marginalised and sampled over as part of a fully Bayesian parameter estimation and model comparison loop, and the Bayesian evidence used to determine/marginalise over the number of components N. The advantage of this is that the approach can reveal systematic effects which are otherwise hidden in a traditional model-dependent treatment. Hints of such features can be seen in the primordial power spectrum as the $\ell \sim 30$, and its presence across both WMAP and Planck indicate that if it is a systematic it can only be in features common to both.

Section b: Methodology

The project begins in Y1 with three work packages aimed at Theory, LFI and Boltzmann solvers. In Y2, we begin applying these techniques to the curvature tension, bring further resource and research into more speculative long-term enhancements to LFI, as well as complementing this strategy with research into model-independent cosmological reconstructions. In Y3, the focal point of the project, the final PDRA joins, and we will aim to have a first attempt at resolving tensions using all the techniques in tandem. PDRA3 will remain until the end of the project, refining and iterating the procedures, and other projects transition to more forward-looking elements and forecasts.

Objectives

(O1) Develop an RKWKB Boltzmann solver
(O2) Distribute the community-standard LFI tools and code
(O3) Create and develop ambitious tension-resolving theories
(O4) Consistently and coherently test alternatives to ΛCDM
(O5) Resolve the curvature tension
(O6) Resolve the Hubble tension

Deliverables

(D1) Multivariate RKWKB code
(D2) Next-generation Boltzmann software
(D3) LFI cosmological software packages
(D4) Cosmological reconstruction framework
(D5) Transdimensional & LF nested sampling
(D6) A new model of the Universe

Dissemination

Other than the potential for a solution to (m)any of the cosmological tensions, this project will leave a legacy of techniques and software suitable for future generations of astrophysical researchers. I have already fostered a culture within my PhD students of industry-standard open source software distribution using git(hub) version control, continuous integration, and pip-installable Python packages, for example `maxsmooth` [65, 66], `globalemu` [67], `anesthetic` [33], `oscode` [47, 68]. I also maintain the highest data management practices, ensuring that with any publication, the data, code and plotting scripts are reproducibly made available on Zenodo [69–71]. This research hygiene ensures the products are both available to and able to be challenged by future researchers (including most importantly ourselves). As with all my research, papers produced by the project will be made openly available for free on arχiv, in addition to being published in a range of journals such as MNRAS, PRD, JCAP, JHEP, Nature (astronomy) and PRL. We will further publicise our work at seminars, workshops and conferences. Two collaborations are involved in the proposal:

GAMBIT, the Global and Modular Beyond-standard-model Inference Tool is an open-source software supported by an international community of ≈ 60 particle physicists, cosmologists and statisticians. It provides the only common interface for particle physics and cosmological data [72], and is under rapid and active development. It is an open community, with a contribution-based authorship framework and code of conduct, for which all members of the project will be encouraged to join. It is anticipated that the end location for all deliverables will be instantiated, and/or ported to the GAMBIT framework for future researchers to use.

REACH, the Radio Experiment for the Analysis of Cosmic Hydrogen is a collaboration of 40 researchers based in Cambridge and Stellenbosch University, with other members around the world. It is a 21-cm "global" (sky averaged/monopole) experiment, aiming to map the epoch of reionisation and provide constraints on the period of cosmic history known as the dark ages. I lead the Bayesian data analysis team, and there will be opportunities for members of the group to contribute to or become full members of REACH as needed. The most direct interaction will be with complementary constraints on τ the optical depth of the CMB to reionisation.

Risk mitigation

- A significant risk is that we may be unable to find a solution without future datasets. However, even in the event that we do not reveal a successor cosmology to ΛCDM, the techniques, theories and software we build will still provide critical tools otherwise unavailable for future researchers for resolving this most fundamental of cosmological issues. The later stages of the project focus on forecasts to partially mitigate this risk.
- In the event that the computing power requested is not sufficient to carry out some of our more expensive analyses, we can obtain time from GAMBIT community resources, from the DiRAC resource allocation I currently hold, and apply for future follow-on DiRAC resource.
- Another "risk" is that the project could reveal that all the current tensions are explained by systematics rather than new physics. Whilst this would be scientifically disappointing, this would still be a significant step forward for the community, allowing us to proceed on a solid standard model foundation as more statistically powerful datasets are acquired.

WP-A Next-generation Boltzmann solvers [PHD1]

This work package aims to fundamentally change the way we perform the calculations at the heart of the Boltzmann codes, by using proven techniques developed by my group [46–50] for solving differential equations whose solutions move between oscillatory and frozen/slowly changing modes. It will be conducted by PHD1 under close supervision by myself, and WP-F, WP-D, WP-G have dependency on some products of this work.

The primary objective (O1) of this work package is a mathematical technique, and publicly available set of software (D1) & (D2), capable of solving Boltzmann equations for perturbations numerically for every mode without making approximations at speeds comparable to existing approaches.

The current RKWKB approach allows the efficient solution of $x(t)$ satisfying the linear differential equation

$$\ddot{x}+2\gamma(t)\dot{x}+\omega^2(t)x=F(t) \tag{3}$$

for the case where $F=0$ (unforced), where γ and ω are time-dependent friction and frequency terms. This tried-and-tested approach [47, 49, 50] was developed and distributed in detail by one of my PhD students [68] under my direction, and is now being adopted by other communities [51]. The extension to $F(t)\neq 0$ is an important but straightforward starting project for a PHD1, which will be ideal for them to get to grips with the current state-of-the-art material (OA1).

The larger, most risky part of the project is extending this to the multivariate case (OA2). Previous attempts [50] to generalise this to the multivariate case have begun by considering the first order vector generalisation of this $\dot{\mathbf{x}}=\mathbf{A}(t)\mathbf{x}$. However, one insight which is ripe for exploration is that the success of RKWKB approaches hinge on the second order nature of eq. (3). Promoting x and F to vector valued functions, and γ and ω to matrices gives us access to vector-based WKB techniques in a way that the first order formulation does not. In the event that this is not a profitable line of enquiry, the preliminary research begun in [50] leads to several more “safer” but engineering-type solutions which can be developed to solve the same problem, so I am confident that (OA2) has a high probability of success in some form.

Once a mathematical strategy has been established and tested, (OA3) aims to package this in a distributable, usable form (D1). Both the method and the code will be peer-reviewed by a traditional journal and JOSS review respectively. The aim is for this to be released at the start of Y2, ready for use by other members of the community and other project participants (who would of course have access to it prior to release for both early use and bug testing).

This technique will then be extended to be applied to write a full Boltzmann solver in order to achieve (OA4) & (OA5) and deliver (D2). The marker of success will be if we can produce dense CMB power spectra without approximations in a similar time to the existing state of the art at equivalent accuracy, with secondary markers being improved accuracy or speed relative to these. A first draft (but not necessarily publicly released) version of this code will need to be achieved by the start of Y3 in time for the heart of the project, where this will contribute to WP-G.

Through Y3, it is expected that the PhD student will be on hand to adjust and refine the code, working in parallel with the other postdocs and students, which will obviously involve refining the code. In parallel the student will start porting the code into community tools such as GAMBIT and cobaya (and would be expected to join the GAMBIT community). Y4 will be a continuation of the above projects, code maintenance, writing thesis and applying for jobs (either academic or industrial).

Over the course of the PhD project I would expect the student to publish at least three first-author papers, one on the mathematical techniques underlying the methods, one as a full software release with cosmological examples, and one on testing novel and challenging cosmological models such as frozen initial conditions, likely in conjunction with PDRA1 and PHD2.

(OA1) Extend theory and state of the art to include forced equations.
(OA2) Extend theory and state of the art to include multivariate RKWKB
(OA3) Develop performant, publicly distributable code for solving multivariate differential equations
(OA4) Build Boltzmann solver on top of this, demonstrating precision adjusted speed-up on standard model
(OA5) Apply this solver to extensions such as curvature and neutrinos, demonstrating equivalent speed up
(OA6) Incorporate this code into community tools such as GAMBIT and cobaya

WP-B Novel tension-reducing theory [PHD2]

The aim of this work package is to train a dedicated theorist in time for Y3 who is capable of quickly utilising the results of other the work packages to select, modify and propose a new paradigm for cosmology. I have a track record of developing several novel and innovative cosmological theories in collaboration with students [25, 73–76], and so have chosen several concrete theoretical projects which I believe show promise for resolving cosmological tensions. As with any theoretical project however, both I and the student will need to be prepared to pivot if new community directions arise with tension resolving properties which are liable to be proved or disproven by our data analysis strategies. This work package will therefore proceed on several fronts and but shall be reactive depending on which theories prove effective at resolving tensions.

The primary goal of this work package (O3) therefore will be the proposal of a set of novel cosmological theories (in addition to the current state of the art) capable of resolving one or more of the tensions in cosmology, which can be put to the test by other members of the team in WP-G. While PHD2 familiarises themselves with the literature and modern cosmological techniques (OB1), which in practice takes roughly a year with guidance, they will also begin to undertake the three following projects in the order (or threading) which they see fit:

(OB2) Poincaré gauge theory gravities [73, 77, 78]. These have the desiderata of resolving (a) the Hubble tension (through an effect which phenomenologically appears as an early dark energy effect), (b) the dark tension, by replacing both dark energy and dark matter with gravitational effects, and (c) go some way to addressing the quantum gravity tension through weak renormalisability criteria. The project will build on the work of Will Barker, who will be holding a Girton College Research fellowship until Y3 of the project, who will be able to provide additional advice and mentorship if desired. In particular the main goal will to be to build the perturbation theory necessary to test these theories against data, which has strong overlap with WP-A.

(OB3) Initial conditions tension. Efstathiou & Gratton [79] have made clear the tension, which states that a traditional inflationary set-up is predictive of a flat universe. What therefore should we make of the result that the CMB prefers a curved universe? Does this mean that we need a different theory of inflation, such as finite inflation? If so, how does this impact our existing understanding of natural inflation, and how might this impact on theoretical questions of measures on the multiverse? There is also a wealth of research asking to what extent these initial conditions are constrainable using the cosmic microwave background from WP-D, beginning with the work of [25, 80]

(OB4) There has been a recent profitable line of research into theories of the Future Conformal Boundary [74]. This was begun by Anthony Lasenby and myself, and I have been leading efforts in recent years to test these theories against data [25, 81]. These theories consider the global structure of spacetime, and the impact that this has on the mathematical analysis of perturbations (as an analogy, think of the fact that Fourier modes are quantised on a sphere). It turns out that the future conformal boundary at $t = \infty$ of a universe containing dark energy induces an analogous quantisation of wavevectors. This modifies the calculations we make for predicting what we see in the CMB, and has an impact on these theories with potential Hubble tension resolving properties. Investigating these theories further both numerically and practically has been shown to require precisely the Boltzmann solvers from WP-A [25, 49].

There is a small risk that the recruited PHD2 may end up not be up to the challenge, in which case more of the theoretical work will have to be undertaken by other members of the team, and the student set more clear numerical/computational tasks. Since the University of Cambridge has extremely strong PhD applicants, I however view this scenario as rather improbable.

In addition, it is expected that the student will work with PHD1 in helping on the theoretical side of Boltzmann solver implementations for WP-A and deliverable (D2). It is expected that a talented student will be able to produce three to five first-author papers over the course of their PhD on such theoretical topics. In addition, the student will heavily contribute to the theoretical elements of the data analysis efforts.

(OB1) Familiarise with current state of the art in tension resolving theories
(OB2) Develop and apply perturbation theory for Poincaré gauge theory gravities
(OB3) Investigate initial conditions for inflation tension
(OB4) Develop future conformal boundary theory for tension resolution

WP-C Compromise-free likelihood-free inference [PDRA1]

One of the major goals of this project is to establish likelihood free inference as a reusable and trusted tool for cosmological analysis (O2). The primary issue with likelihood free inference as it stands is its dependency on machine-learning and optimisation-based training, and this ambitious project seeks to release a form of likelihood free inference which is robust and trustable in the long term, delivered as a package of software tools with interfaces to a variety of languages such as Python, Julia, C++ & Fortran (D3).

In this work package, we will develop and apply the field of "Compromise-free likelihood-free inference". This approach combines my work on Bayesian sparse reconstructions [63], Bayesian neural networks [82], and the current community standards [54, 55].

The first objective will be achieved within Y1, that of a version 1.0.0 of an industry standard tool for using likelihood free inference in a cosmological setting which can be built upon (OC1). The remainder of the work package will focus on applying the tool to several cosmological examples of importance, namely data from Planck CMB (OC6), and Planck lensing (OC2), BAO (OC3), supernovae (OC4), and weak lensing (OC5) both to the other work packages and to the wider cosmological community.

For an experienced postdoc, it is expected that these projects will take approximately six months each, but taken in an order most suited to their skill set (as it will be easier to begin with the field in which they have most experience). This set of tools will have impact on several other work packages, so the PDRA starts in the first year in order that by Y3 the tools are mature and able to be applied to a variety of other situations.

The key idea behind CFLFI is to replace maximisation training with marginalisation training. Given a proxy for a distribution $f(\theta,D,w)$, where this may be a density estimator for the likelihood, posterior or joint distribution, at the moment the current strategy is to determine the weights w by optimisation. In the case of DELFI, this is done by optimising the weights of a neural network density estimator, and in the case of BOLFI this is by Bayesian optimisation. In Bayesian sparse reconstruction [63], we have shown that in machine learning it is far more robust to train machine learning hyperparameters by marginalisation/sampling rather than optimisation (as any Bayesian should instinctively feel).

At its heart, we identify the proxy distribution f with a probability conditional on its parameters w. This means that we have a well-defined likelihood, which we may invert and marginalise/sample over as usual:

$$L(w) = \prod_i f(\hat{D}_i,\theta_i|w) \xrightarrow{\text{Bayes theorem}} P(w|\{\hat{D},\theta\}) \xrightarrow{\text{marginalisation}} f(D,\theta) \approx \int f(D,\theta,w)P(w|\{\hat{D},\theta\})dw \quad (4)$$

which we may confidently numerically perform using standard techniques such as nested sampling. This would apply for any proxy f, be it a neural density estimator, a Gaussian process or a Gaussian mixture model, and this project will explore all of these, and provide a generalised framework package for implementing any of them (D3). One can of course also use Bayesian evidences to choose between the best proxy, or indeed to marginalise over them.

For each of the data-based work packages, the aim will be to reproduce the standard results from each analysis. Some of these has been performed in the literature using DELFI or BOLFI [54, 59, 83], and the aim will be to verify or contest both these and the community accepted result.

The main risk with this project is the ambitiousness of number of examples that may be achievable within three years. If CFLFI proves to be as robust and generalisable as it promises, then this timeline proves the efficacy of the approach. In the event that it is not possible to fulfil all six within a short space of time, it is likely that other team members could take over, or support. However, if even a subset of these analyses are completed this will be of great value to the community

(OC1) Release a reusable code for compromise-free likelihood-free inference
(OC2) apply CFLFI to CMB lensing and compare to the standard result
(OC3) apply CFLFI to BAO and compare to the standard result
(OC4) apply CFLFI to Supernovae data and compare to the standard result
(OC5) apply CFLFI to weak lensing and compare to the standard result
(OC6) apply CFLFI to CMB and compare to the standard result

WP-D Cosmological reconstructions on present and future data [PHD3]

The goal of work package is to systematically reconstruct elements of cosmic history in a model-agnostic fashion, with a view to uncovering or sharpening existing tensions and revealing systematic errors. The inclusion of a suite of current and upcoming large-scale-structure datasets will also prove critical in simultaneously reconstructing the late-time elements of cosmological models. More generally the work package aims to further build expertise within the group for working with the full spectrum of available up-to-date and future forecasted datasets, in time for WP-G.

The work package will begin with the student setting up a generalised pipeline for Bayesian cosmological reconstruction (OD1). Success will be determined by a reproduction of the results in [60] for primordial power spectrum reconstruction with CMB data, and a version 1.0.0 of (D4).

Once that start point is established, the project will move on to apply the same techniques to a variety of other elements of cosmic history (OD2), for example the inflationary potential $V(\phi)$ [60], the dark energy equation of state $w(z)$ [61, 62], reionisation history $x_e(z)$ [84], late-time matter power spectrum $P(k)$, CMB power spectra C_ℓ and CMB lensing spectra $C_L^{\phi\phi}$. Some of these have been performed in the literature before using other techniques, whilst others later in that list would be genuinely novel, but a unique feature of these methods is that they are capable of performing these reconstructions by solving the full simultaneous high-dimensional problem without approximation.

A key question of interest from this project, aiming to be answered in Y3 (second year of PHD3) will be whether datasets which in theory should be able to reconstruct similar functions (such as Planck, ACT and SPT) are consistent in their reconstructions [85]. This would correspond to a novel field of functional tension quantification (OD3), and could prove critical for locating systematic errors or differences between independent and/or correlated datasets.

In parallel to these, with a view to improving the efficiency of these approaches for present future users, the student will also investigate transdimensional nested sampling methods [86] (OD4). The aim here is to treat the number of reconstruction variables N as a parameter that is also varied in the sampling. At the moment, the closes one comes to this is the "adaptive" method [61–63, 87], but this has several drawbacks, the principle one bing that one must specify a maximum value for N, with a consequent penalty in speed that is of order N_{max}. Transdimensional nested sampling [86] only adds in parameters on demand and therefore forms a more compact and adaptive generalisation of the approach. For single reconstructions this is a mere convenience, but it will prove essential if one wishes to reconstruct more than one unknown function.

The final aim of this work package (OD5) therefore will be to use the advanced sampling techniques I have developed [30, 31, 34, 61] to perform simultaneous reconstructions of these quantities, such as $x_e(z) + \mathscr{P}_\mathscr{R}(k)$, $(A_k, B_k) + V(\phi)$ and $P(k) + w(z)$. By virtue of their ability to accommodate simultaneous changes to different aspects of cosmological modelling, these reconstruction techniques are capable of revealing the underlying physical cause of the Hubble tension. As transdimensional nested sampling will prove invaluable to future researchers beyond the context of reconstructions [63], packaging the transdimensional nested sampling methods into a reusable code for others will be a key deliverable of this research programme (D5).

Toward the end of the PhD and research programme, focus will shift toward forward-looking analyses by moving to forecast data such from the SKA, Euclid and CMBS4 [88]. The aim here will be to determine the extent to which future datasets will be able to shed light further on any existing tensions, informed by the work from earlier in the project. Building these forecast pipelines will also be useful for further de-risking the project, as in the event that we are unable to uncover the cause of cosmological tensions using present day data, we will be able to answer questions with regard to future experiment's potential. In the event that present-day data are insufficiently resolved to full achieve our goals, we will validate our methodologies on simulated datasets (OD6).

(OD1) Develop a generalised reconstruction pipeline
(OD2) Reconstruct separate elements of cosmic history using present day data
(OD3) Use functional tension quantification techniques to determine where tensions reside
(OD4) Develop transdimensional nested sampling in the context of reconstructions
(OD5) Apply transdimensional nested sampling to simultaneous reconstruction
(OD6) Determine the future limits and potential of these techniques on forecast data

WP-E Likelihood-free nested sampling and Bayesian machine learning [PHD4]

This work package has a degree of cross-talk with WP-C, and there are two years of crossover (Y2&Y3) between PDRA1 and PHD4 to ensure that a fully skilled PDRA is able to transfer their experience and know-how to PHD4. As a result a presence of LFI expertise is kept throughout the project for all other work packages.

This work package is more forward-looking toward the future of inference and LFI, and comprises more speculative research which will complement and enhance the work of PDRA1. It is therefore higher risk, but also higher reward, and far more suitable to an open-minded PhD student than a seasoned PDRA. It will complement the other work packages and will go some way to achieving one of the broadest aims (O2) and key deliverables (D3) of this project, whilst simultaneously improving the robustness of the use of LFI for analysing and relaxing tensions.

The first objective is to make a stronger link between nested sampling and likelihood free inference (OE1), advance the field of likelihood free nested sampling and apply it to the same cosmological examples drawn from WP-C. Some headway has been made into this field in the context of systems biology [89] using a likelihood approximation via particle filters to good effect, releasing this code as (D5). The first project for the PhD student will be to replicate the results in this paper in the context of cosmology, before replacing particle filters with a more general and principled mechanism for density estimation. Particular emphasis will be placed on using the fast-slow mechanism for mock data generation and marginalisation which has been so successfully demonstrated elsewhere in cosmology [90].

The second objective (OE2) is to improve on the state of the art in data compression, currently represented by [58]. This is a non-linear generalisation of MOPED [57], but is still based on Gaussian approximations by necessity. The aim here will be to bring the advances the group has made in high-dimensional nested sampling to bear on the problem of data compression. Extreme data compression is an essential part of likelihood free inference at the moment, and the broad scope of this second objective will be to incorporate adaptive data compression into the likelihood free inference framework (D3).

Alongside this in (OE3), we will explore the emerging field of topological data analysis [91, 92] (TDA). This is an approach for extracting information from datasets that are high-dimensional, incomplete and noisy, by focussing on topological properties (or equivalently compressions) of the data, such as persistence homology and Betti numbers. To my knowledge, nobody has yet made a connection between LFI and TDA.

Much of the success in recent advances in LFI in cosmology has been through the use of neural density estimators. Despite their expressiveness, the critical drawback is the requirement of manual tuning (or "magic hands") on the part of the researcher. One way around this is to let Bayes theorem do the tuning in the context of compromise-free Bayesian Neural Networks [82] through `PolyNet` (OE4)

Data compression is one of the key bottlenecks in the LFI pipeline, however a significant orthogonal challenge in both nested sampling and likelihood free inference is one of dimensionality, so (OE5) aims to advance the field of accelerated nested sampling. Nested sampling has excellent dimensionality scaling, but in practical problems much of the cost of a nested sampling run is in "burning in" from prior to posterior. Posterior repartitioned nested sampling [93] gives a way to drastically speed up this portion of the algorithm and hence access far higher dimensionalities. It is akin to the manner in which pre-trained proposal covariance matrices can be used to speed up the convergence of a Metropolis Hastings run. Implementing this would allow an apples-with-apples comparison of speed between the two approaches. The other approach is reversible nested sampling, whereby one begins at a known peak and reverses into the posterior, which is typically a shorter distance statistically speaking than from the prior. Both of these would provide a way of using a cheaper approach such as maximisation or Markov Chain Monte Carlo to dramatically speed up nested sampling and give access to higher dimensional problems.

The PhD will complete with the release of these advances as part of our LFI package initially developed in WP-C for (OC1) & (D3).

(OE1) Bring likelihood free inference + nested sampling up to the state of the art in both
(OE2) Develop Bayesian Kullback–Leibler-based simultaneous compression statistics
(OE3) Explore the use of Topological data analysis in LFI compression
(OE4) Investigate the use of Bayesian Neural Networks (`PolyNet`) as a Bayesian neural density estimator
(OE5) Use accelerated nested sampling for higher-dimensional CFLFI
(OE6) Release next-generation version of LFI for cosmology as codebase

WP-F Resolve curvature tension [PDRA2]

The Planck 2018 release reported a cosmic microwave background consistent with the concordance ΛCDM model, with no evidence for extensions beyond this. This is summarised in Table 4 of [94]. There is however a hidden story in this, in that the table shows the Planck CMB data TT,TE,EE+lowE preferring a closed universe at more than 95% confidence. Adding in geometrical degeneracy breaking likelihoods such as CMB lensing and/or BAO data brings the universe squarely back to flat, but one should be careful with the logic here. In my paper [16] I point out that in the context of curvature, CMB lensing and BAO are in tension with Planck, and therefore suspicion should be cast on results derived from their combination. A Nature astronomy paper [17] was subsequently made public with similar conclusions.

The curvature phenomenon is linked to the A_L tension, although it is not fully equivalent. Closed universes have a preference because they adjust the lensing quality of the CMB (in a very similar manner to the phenomenological parameter A_L) and suppress power at low multipoles, which resolves two tensions at once. In spite of the very low H_0 it interestingly does not make the Hubble tension worse since the error bars are increased. A third paper [79] demonstrated that an alternative likelihood with a more ambitious use of the Planck data reduce the strength of these conclusions [79], although tension and a moderately strong preference for closed universes still remains.

Some would argue that admirable conservatism within the part of the Planck collaboration meant that this result was not highlighted as possible evidence of beyond-ΛCDM cosmologies, but it should give us pause for thought before we go so far as to say that Planck unambiguously prefers a flat universe. What can be said without qualification is that using only cosmic microwave background data (without CMB lensing) both Planck likelihoods express a preference for a universe with non-zero cosmic curvature. What remains up for debate in the literature is how much we should be concerned about a $2-3\sigma$ tension.

The critical problem, and part of this proposal's overarching thesis in terms of why the community has struggled to resolve these tensions, is that both BAO and lensing depend on a flat universe assumptions. For CMB lensing, the likelihood is expanded about a fiducial flat cosmology, and the simulations used for calibration are also derived in the context of a flat model [95]. The same is true to a large extent for BAO, whose reconstruction templates, simulations and fiducial cosmology for redshift and clustering all depend on a flat ΛCDM. Given this in-built bias toward ΛCDM it is not surprising that these datasets break the degeneracy with posteriors centred suspiciously close to zero. Until these biases are drawn out of the pipelines some doubt in the community will always remain as to the strength of support for a flat universe. Putative probes such as cosmic chronometers [96] may one day provide an independent technique for measuring curvature although at present these are plagued by assumptions associated with galactic evolution physics which have their own systematic errors which are arguably in part still indirectly inferred from a flat universe.

A curved universe has substantial implications for almost all the tensions, but it is clear that curvature on its own does not solve the problems, since curved universes whilst not being further in tension due to increased error bars nonetheless have rather improbable values of Ω_m and H_0 given other datasets. Some headway has been made into exploring curvature $\pm X$, where X is another adjustment to the model [97], but in the same way as for the Hubble tension, until we have ironed out all potential systematic errors in other likelihoods it is hard to draw any conclusion with certainty. More importantly, curvature already tends to push Boltzmann codes toward their limits of speed and accuracy, so adding an additional $\pm X$ complexity means without the next-generation solvers from WP-A there will always be somewhere for flat universe sceptics to hide.

The aim of this work package is to close the book once-and-for-all on the curvature question. If upon removing these residual assumptions from the BAO pipeline one still finds it collapses to flat, then this is suggestive that the cause may be statistical fluctuation or another A_L-like systematic. If adjusting these biases one finds that BAO prefer a curved universe, then that has Nature-worthy implications for all cosmology.

This is a challenging project requiring advanced skills and knowledge of CMB and BAO pipelines, and hence is suitable for a postdoctoral researcher over several years, rather than a PhD student. Its primary aim is encapsulated in (O5) which will partially address (D6).

(OF1) Remove residual flat assumptions from baryon acoustic oscillation pipeline
(OF2) Remove residual flat assumptions from CMB lensing pipeline
(OF3) Test CMB, BAO and lensing in the context of curvature
(OF4) Investigate curvature $\pm X$ models

WP-G Combining diverse data for complete tension resolution [PDRA3]

This work package begins in year three, at the heat and heart of the research programme. All eight participants are present and to a large degree fully skilled. The role of this PDRA and work package will be to bring together all the threads (in collaboration with all participants) and carry through until project end.

The primary objective of the work package is to combine diverse datasets in a consistent likelihood-free driven framework with the aim of revealing the true cause of the tension, pruning away any data with systematic errors and using Bayesian model comparison to select the next standard model of cosmology (O6) & (D6).

The Likelihood-free inference products from WP-C and WP-E will be used for disentangling systematics. The Boltzmann codes from WP-A will mean that we are capable of testing extensions without standard model bias. The novel combination of theories proposed by WP-B and WP-F will form our universe of models for explaining the tension, which will in turn have been informed and crafted by the model independent analyses of WP-D. The first objective (OG1) will be to establish this in a single distributable and replicable framework (D3).

The second objective (OG2) once all this is in place toward the end of Y3 will be to make a first attempt at determining which (if any) of the models is capable of resolving all the tensions in the (systematics adjusted/pruned) data. It is likely that this will require iteration, which is the primary reason this primary output of the project has an aim at the temporal centre rather than the end.

In addition to this, we aim to bring into our framework novel combinations of data using the GAMBIT collaboration and tools (OG3). In particular we intend to use particle physics likelihoods from GAMBIT to constrain theories of dark matter and neutrinos with more precision, as well as to gain access to likelihoods as varied as gravitational waves, the Cherenkov telescope array (CTA) and the Large Hadron Collider (LHC), all of which are present already or are planned to be incorporated.

We also aim to use the REACH collaboration's global 21-cm both public results and (as members of the collaboration) in-progress pipelines to generate novel optical depth τ constraints (OG4) [66, 98, 99]. This is particularly critical, as 21-cm global experiment provide access to more precise modelling of reionisation and an alternative method of measuring τ, which is one of the rate-limiting parameters in CMB datasets.

The project finishes using the experience from WP-D to apply the same techniques to forecast data (OG5), using 21-cm from SKA, weak lensing from Euclid, CMB from SO and CMBS4. This is an important next step whether or not we resolve the tension, since it will be in the not-too-distant future beyond project end in which these data sets start being delivered, and the pipelines we have developed will be able to be applied to this next onslaught of big data.

It is my belief that even when the systematics are disentangled from the new physics, no single theoretical solution will prove capable of satisfactorily resolving all the cosmological tensions. I anticipate that the true solution will be a combination of changes, for example, removing dark energy and replacing it with modified gravity, or adding in curvature and adjusting neutrinos. It is also important to note the cognitive biases [100] with regard to a human preference for additive rather than subtractive modifications to a model, which an automated Occam's razor (i.e. Bayesian model comparison) rectifies. However, until all the moving parts (theoretical, analytical and computational) of this project are in place, it would not be prudent to purely theorise as to the nature of the next paradigm for precision cosmology. Only the data can tell.

(OG1) Synthesise all prior work package products into a coherent statistical framework
(OG2) Taking a global view, determine which (if any) model is capable of consistently fitting all the data.
(OG3) Incorporate GAMBIT particle physics data for constraining neutrino and dark matter properties.
(OG4) Use REACH 21-cm data for improved τ constraints
(OG5) Apply the same framework to forecasted data

Y1
Y2
Y3
Y4
Y5
[PI 50%]
WP-A [PHD1]
(OA1)
(OA2)
(OA3)
(OA4)
(OA5)
(OA6)
WP-B [PHD2]
(OB1)
(OB2)
(OB3)
(OB4)
WP-C [PDRA1]
(OC1)
(OC2)
(OC3)
(OC4)
(OC5)
(OC6)
WP-D [PHD3]
(OD1)
(OD2)
(OD3)
(OD4)
(OD5)
(OD6)
WP-E [PHD4]
(OE1)
(OE2)
(OE3)
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